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High-Frequency Sum-Rules for Classical Relativistic Plasmas in a Magnetic Field

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High-frequency sum rules for the transverse elements of classical relativistic plasmas in a magnetic field are derived. The relativistic effect reduces the plasma mode frequency by a factor of $\langle\gamma^{-1}(1-v^2/3c^2)\rangle$.

1. INTRODUCTION

In the derivation of the high-frequency sum rules of the full dielectric tensor (Kalman and Genga, 1986; Genga, 1988*a*) based on a nonrelativistic approach, we found that the inclusion of the transverse fields led to the appearance of contributions of order v^2/c^2. To achieve a consistent treatment it would be of interest to calculate the classical relativistic high-frequency sum rules using a relativistic method.

The known results pertain to the full dielectric tensor in an isotropic situation (Genga, 1988*b*). It is known (Kalman and Genga, 1986) that in an anisotropic system in the presence of an external magnetic field, the dielectric tensor has six independent elements and this makes the problem more complicated. Further, the relationship between the elements of the external and current–current response function elements and the elements of the dielectric tensor become quite involved. Finally, the appearance of the cyclotron frequency renders the structure more complex.

In this work, therefore, I study the high-frequency behavior of the full dielectric tensor in an anisotropic situation based on a relativistic approach. I derive the relativistic high-frequency sum-rule expansion up to order ω^{-5}. The method of derivation relies on the Hamiltonian formalism (de Gennes, 1959). As in the previous cases (Kalman and Genga, 1986; Genga, 1988*a*,*b*), the particle Hamiltonian is enlarged so as to include the photon degrees of

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freedom; this allows the description of the transverse interaction. In this situation it is known (Kalman and Genga, 1986; Genga, 1988*a,b*) that in addition to the particle contribution to the sum-rule coefficients, the photon gas, coexistent with the high temperature, generates its own contribution. It is known that the exact evaluation of this contribution is hampered by two conditions: Thus, (1) the classical ultraviolet divergence requires that even within the framework of a classical theory one describe the photons via the quantum Bose–Einstein distribution, and (2) the equilibrium description implies that one single temperature exists for the combined particle–photon system.

In Section 2, I describe the general relations between the external or current–current response function sum-rule coefficients and those of the dielectric tensor. Then the exact ω^{-2}, ω^{-3}, ω^{-4}, and ω^{-5} sum-rule coefficients for transverse elements are calculated. The long-wavelength limit of the result is evaluated in Section 3; its possible applications for the dispersion relation of plasma modes are also discussed in Section 4; the plasma modes in consideration are the extraordinary and the ordinary modes for propagation parallel and perpendicular to magnetic field, respectively. I summarize the results in Section 5.

2. TRANSVERSE SUM RULES

The full dielectric tensor $\varepsilon^{\mu\nu}(\mathbf{k}\omega)$ or the full polarizability tensor $\alpha^{\mu\nu}(\mathbf{k}\omega)$ are expressible in terms of the corresponding "external" quantity $\hat{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ as (Kalman and Genga, 1986; Genga *et al.*, 1988*a,b*)

$$\begin{aligned} \boldsymbol{\alpha} &= \hat{\boldsymbol{\alpha}}(\boldsymbol{\Delta} - \hat{\boldsymbol{\alpha}})^{-1}\boldsymbol{\Delta} \\ \boldsymbol{\Delta} &= \mathbf{11} - n^2\mathbf{T} \\ n &= kc/\omega \\ \mathbf{T} &= \mathbf{11} - \mathbf{k}\cdot\mathbf{k}/k^2 \end{aligned} \tag{1}$$

$\hat{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ is also known to possess the high-frequency sum-rule expansion

$$\hat{\boldsymbol{\alpha}}^{H'}(\mathbf{k}\omega) = -\sum_{\substack{l=1 \\ l\text{ odd}}}^{\infty} \frac{\hat{\boldsymbol{\Omega}}_{l+1}(\mathbf{k})}{\omega^{l+1}} \tag{2}$$

$$\hat{\boldsymbol{\alpha}}^{H''}(\mathbf{k}\omega) = -\sum_{\substack{l=2 \\ l\text{ even}}}^{\infty} \frac{\hat{\boldsymbol{\Omega}}_{l+1}(\mathbf{k})}{\omega^{l+1}} \tag{3}$$

where the superscript H stands for "Hermitian part of" and prime and double prime denote "the real part of" and "the imaginary part of," respectively. I evaluate the $\hat{\Omega}$ coefficients from the relation (Kalman and Genga, 1986; Genga *et al.*, 1988*a,b*)

$$\hat{\Omega}_{l+1}^{\mu\nu}(\mathbf{k}) = \frac{4\pi e^2}{V}\beta_p\left(i\frac{d}{d\tau}\right)^{l-1}\langle j_{\mathbf{k}}^{\mu}(\tau)j_{-\mathbf{k}}^{\nu}(0)\rangle|_{\tau=0} \tag{4}$$

where

$$j_{\mathbf{k}}^{\mu} = \sum_i v_i^{\mu}\exp(-i\mathbf{k}\cdot\mathbf{x}_i) \tag{5}$$

The high-frequency expansions of $\boldsymbol{\alpha}(\mathbf{k}\omega)$ are similar to those of $\hat{\alpha}(\mathbf{k}\omega)$ in equations (2)-(3), with $\Omega_{l+1}(\mathbf{k})$ replacing the corresponding $\hat{\Omega}_{l+1}(\mathbf{k})-s$ (Kalman and Genga, 1986; Genga, 1988*a,b*).

As mentioned in the introduction, the Hamiltonian which takes into account the description of the interaction of the plasma with the transverse electromagnetic field must include the photon degree of freedom. Therefore I have

$$H = \sum_{i=1}^{N}\gamma_i mc^2 + \frac{1}{2}\sum_{i\neq j}V(|\mathbf{x}_i-\mathbf{x}_j|) + \frac{1}{2}\sum_{\mathbf{q}}(\bar{e}_{\mathbf{q}}\bar{\mathbf{e}}_{\mathbf{q}} + q^2c^2\bar{a}_{\mathbf{q}}\bar{\mathbf{a}}_{\mathbf{q}}) \tag{6}$$

where

$$\begin{aligned} \gamma_i &= \left(1-\frac{v_i^2}{c^2}\right)^{-1/2} \\ v_i &= \frac{[\bar{P}_i + i\Lambda\sum_q \bar{a}_{\mathbf{q}}\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{x}_i) - (e/c)A^0(x_i)]/m}{\{1+(1/m^2c^2)[(\bar{p}_i + i\Lambda\sum_{\mathbf{q}}\bar{a}_{\mathbf{q}}\exp(i\mathbf{q}\cdot\mathbf{x}_i) - (e/c)A^0(\mathbf{x}_i)]^2\}} \\ \Lambda &= ie\left(\frac{4\pi}{v}\right)^{1/2} \end{aligned} \tag{7}$$

where x_i, $\bar{P}_i$, $\bar{a}_q$, and $A^0(x_i)$ are the position, momentum, self-consistent field vector amplitude, and external magnetic field vector of ith particle, respectively.

For an anisotropic system, in the presence of external magnetic field, $\hat{\boldsymbol{\alpha}}$ is nondiagonal; hence, both odd and even frequency moments exist. Finally, we consider a coordinate system in which $\mathbf{k}=(0,0,k)$, i.e., B system.

The first moment leads to

$$\begin{aligned} \hat{\Omega}_2^{\mu\nu}(\mathbf{k}) &= \frac{4\pi e^2}{V}\beta_p\langle j_{\mathbf{k}}^{\mu}(0)\mathbf{j}_{\mathbf{k}}^{\nu}(0)\rangle \\ &= \omega_p^2\left\langle\gamma^{-1}\left(1-\frac{v^2}{3c^2}\right)\right\rangle\delta^{\mu\nu} \end{aligned} \tag{8}$$

The second moment leads to

$$\hat{\Omega}_3^{\mu\nu}(\mathbf{k}) = \frac{i4\pi e^2}{2V}\beta_p[\langle \dot{j}_{\mathbf{k}}^{\mu}(0)\mathbf{j}_{\mathbf{k}}(0)\rangle - \langle j_{\mathbf{k}}^{\mu}(0)\dot{\mathbf{j}}_{k}^{\nu}(0)\rangle]$$

$$= \frac{i4\pi e^2\beta}{2}\sum_{ij}\langle(\dot{V}_i^{\mu}V_i^{\nu} - ik^{\alpha}V_i^{\alpha}V_i^{\mu}V_j^{\nu} + ik^{\delta}V_j^{\delta}V_j^{\nu}V_i^{\mu} - V_i^{\mu}\dot{V}_i^{\nu})$$

$$\times \exp[-i\mathbf{k}\cdot(\mathbf{x}_i - \mathbf{x}_j)]\rangle \tag{9}$$

where $\dot{V}_i^{\mu}$ is the acceleration of the ith particle in the μ direction given by

$$\dot{V}_i^{\mu} = \frac{\gamma_i^{-1}}{m}\left(\delta^{\mu\alpha} - \frac{V_i^{\mu}V_1^{\alpha}}{c^2}\right)\left\{-\frac{\partial\Phi}{\partial x_i^{\alpha}} - i\Lambda\sum_{\mathbf{q}} e_{\mathbf{q}}^{\alpha}\exp(i\mathbf{q}\cdot\mathbf{x}_i)\right.$$

$$\left. + \Lambda\sum_{q}[\mathbf{V}_i\times(\mathbf{q}\times\bar{\mathbf{a}}_{\mathbf{q}})]^{\alpha}\exp(i\mathbf{q}\cdot\mathbf{x}_i) + 2\frac{e}{c}(\mathbf{V}_i\times\mathbf{B}^0)^{\alpha}\right\} \tag{10}$$

where

$$\Phi = \frac{1}{2}\sum_{i\neq j} V(|x_i - x_j|)$$

$$\frac{\partial\Phi}{\partial x_i^{\alpha}} = \frac{\partial H}{\partial x_i^{\alpha}} + \Lambda\sum_{i}\sum_{\mathbf{q}} a_{\mathbf{q}}^{\alpha'}v_i^{\alpha'}q^{\alpha}\exp(i\mathbf{q}\cdot\mathbf{x}_i) + \frac{e}{c}v_i^{\alpha'}\frac{\partial A^{0\alpha'}}{\partial x_i^{\alpha}}(x_i) \tag{11}$$

Thus, equation (9) reduces to

$$\hat{\Omega}_3^{\mu\nu}(\mathbf{k}\omega) = \frac{i\omega_p^2 e}{mc}\left\langle \gamma^{-2}\left(1 - \frac{2v^2}{3c^2}\right)\varepsilon^{\mu\nu\rho}\right\rangle B^{0\rho} \tag{12}$$

third moment yields

$$\hat{\Omega}_4^{\mu\nu}(\mathbf{k}) = \frac{4\pi e^2}{V}\beta_p\langle \dot{j}_{\mathbf{k}}^{\mu\nu}(0)\dot{\mathbf{j}}_{\mathbf{k}}^{\mu\nu}(0)\rangle$$

$$= \frac{4\pi e^2}{V}\beta_p\sum_{ij}\langle(\dot{V}_i^{\mu}\dot{V}_j^{\nu} + k^{\alpha}k^{\delta}V_i^{\alpha}V_i^{\delta}V_i^{\mu}V_i^{\nu}$$

$$- ik^{\alpha}V_i^{\alpha}\dot{V}_j^{\nu}V_i^{\mu} + ik^{\alpha}\dot{V}^{\mu}V_j^{\delta}V_j^{\nu})$$

$$\times \exp[-i\mathbf{k}\cdot(\mathbf{x}_i - \mathbf{x}_j)]\rangle \tag{13}$$

It is also known (Kalman and Genga, 1986; Genga, 1988*a,b*) that the presence of the photon degrees of freedom $a_{\mathbf{q}}^{\mu}$, $e_{\mathbf{q}}^{\mu}$ gives rise to averages of field coordinates of the form $\langle a_{\mathbf{q}}^{\mu}\mathbf{a}_{\mathbf{q}}^{\mu}\rangle$ and $\langle e_{\mathbf{q}}^{\mu}\mathbf{e}_{\mathbf{q}}^{\mu}\rangle$. In strict thermal equilibrium these are expressible in terms of inverse particle temperature β_p and radiation temperature β_r, respectively, and have to be calculated quantum mechanically even in the framework of a classical theory as in this case. By introducing

$$C_q^i = \frac{1}{\sqrt{2}}[\omega_{\mathbf{q}}^{1/2}a_{\mathbf{q}}^{\mu} + \frac{i}{\omega_{\mathbf{q}}^{1/2}}e_{\mathbf{q}}^{\mu})\varepsilon_{\mathbf{q}}^{\mu i}$$

$$\omega_{\mathbf{q}} = qc \tag{14}$$

as a new set of coordinates with polarization vectors $\varepsilon_{\mathbf{q}}^{\mu i}$, and

$$\eta_{\mathbf{q}}^{i} = C_{\mathbf{q}}^{i*} C_{\mathbf{q}}^{i} \tag{15}$$

identified as the equivalent of the photon number operator, then averages are obtained by setting (Kalman and Genga, 1986; Genga, 1988*a,b*)

$$\langle \eta_{\mathbf{q}}^{i} \rangle = [\exp(\beta_r \hbar \omega_{\mathbf{q}}) - 1]^{-1}$$

Thus, equation (13) becomes

$$\hat{\Omega}_4^{\mu\nu}(\mathbf{k}) = \omega_p^4 \Big\langle \gamma^{-2}\Big(1 - \frac{2v^2}{3c^2}\Big)\Big\{ \delta^{\mu\nu} + \frac{\beta_p}{\beta_r} T_{\mathbf{k}}^{\mu\nu}[f(\mathbf{x}_{\mathbf{k}}) - 1] + \frac{k^2}{\kappa^2}(3L_{\mathbf{k}}^{\mu\nu} + T_{\mathbf{k}}^{\mu\nu})$$
$$+ \frac{1}{N}\sum_{\mathbf{q}} L_{\mathbf{q}}^{\mu\nu}(S_{\mathbf{k}-\mathbf{q}} - S_{\mathbf{q}}) + \frac{\beta_p}{\beta_r}\frac{1}{N}\sum_{\mathbf{q}} T_{\mathbf{q}}^{\mu\nu} S_{\mathbf{k}-\mathbf{q}} f(x_{\mathbf{q}})\Big\}\Big\rangle \tag{16}$$

where

$$L_{\mathbf{k}}^{\mu\nu} = \frac{k^\mu k^\nu}{k^2}$$
$$T_{\mathbf{k}}^{\mu\nu} = \delta^{\mu\nu} - k^\mu k^\nu / k^2$$
$$x_{\mathbf{k}} = \hbar \omega_{\mathbf{k}} \beta_r \tag{17}$$
$$f(x) = \frac{x}{e^x - 1}$$

In order for (16) to be acceptable, we argue as before (Kalman and Genga, 1986; Genga, 1988*a,b*) that in the limit $k \to 0$ the distinction between particle (longitudinal) and radiation (transverse) temperatures is meaningless, hence β_r is treated as a k-independent quantity, such that $\beta_r(k=0) = \beta_p$; but $\beta_r(k \neq 0)$ is unaffected by this condition.

The fourth moment yields

$$\hat{\Omega}_5^{\mu\nu} = \frac{i4\pi e^2 \beta_p}{2V} \sum_{ij} [\langle \ddot{j}_{\mathbf{k}}(0) \dot{\mathbf{j}}_{\mathbf{k}}(0)\rangle - \langle \dot{j}_{\mathbf{k}}(0) \ddot{\mathbf{j}}_{\mathbf{k}}(0)\rangle]$$
$$= \frac{i4\pi e^2 \beta_p}{2V} \sum_{ij} \langle\langle (\ddot{V}_i^\mu \dot{V}_j^\nu + ik^\beta V_j^\beta V_j^\nu \ddot{V}_i^\mu - ik^\alpha \dot{V}_i^\alpha \dot{V}_j^\nu V_i^\mu$$
$$+ k^\alpha k^\delta \dot{V}_i^\alpha V_i^\mu V_j^\delta V_j^\nu + 2k^\alpha V_i^\alpha \dot{V}_i^\mu \dot{V}_j^\nu$$
$$+ 2k^\alpha k^\delta V_i^\alpha V_j^\delta V_j^\nu \dot{V}_i^\mu - k^\alpha k^\gamma V_i^\alpha V_i^\gamma V_i^\mu \dot{V}_j^\nu$$
$$+ k^\alpha k^\gamma V_i^\alpha V_i^\gamma V_i^\mu V_j^\delta V_j^\nu - \dot{V}_i^\mu \ddot{V}_j^\nu + ik^\delta V_i^\delta V_i^\mu \ddot{V}_j^\nu$$
$$- ik^\alpha \dot{V}_j^\alpha \dot{V}_i^\mu V_j^\nu - k^\alpha k^\delta \dot{V}_j^\alpha V_j^\nu V_i^\delta V_i^\mu + 2k^\alpha V_j^\alpha \dot{V}_j^\nu \dot{V}_i^\mu$$
$$- 2k^\alpha k^\delta V_j^\alpha \dot{V}_j^\nu V_i^\mu V_i^\delta + k^\alpha k^\gamma V_j^\alpha V_j^\gamma V_j^\nu \dot{V}_i^\mu$$
$$- k^\alpha k^\gamma V_j^\alpha V_j^\nu V_j^\gamma V_i^\delta V_i^\mu)\rangle\rangle \tag{18}$$

Using the same arguments as those used in the evaluation of $\hat{\Omega}_4^{\mu\nu}(\mathbf{k})$ above, we find that equation (18) reduces to

$$\hat{\Omega}_5^{\mu\nu}(\mathbf{k}) = \frac{i3\omega_p^2 eB^{0\rho}}{m^2\beta_p c}\langle\gamma^{-5}[(\varepsilon^{\mu\alpha\rho}k^\nu + \varepsilon^{\alpha\nu\rho}k^\mu)k^\alpha + \varepsilon^{\mu\nu\rho}k^2]\rangle$$
$$+\frac{i\omega_p^4 eB^{0\rho}}{2mc}\left\langle\gamma^{-5}\left[4\varepsilon^{\mu\nu\rho} + \frac{1}{N}\sum_{\mathbf{q}}(S_{\mathbf{k}-\mathbf{q}} - S_{\mathbf{q}})(L_{\mathbf{q}}^{\delta\nu}\varepsilon^{\mu\delta\rho} + L_{\mathbf{q}}^{\delta\mu}\varepsilon^{\delta\nu\rho})\right.\right.$$
$$\left.\left.+(2\varepsilon^{\mu\nu\rho} + T_{\mathbf{q}}^{\delta\mu}\varepsilon^{\delta\nu\rho} + T_{\mathbf{q}}^{\delta\nu}\varepsilon^{\mu\delta\rho})S_{\mathbf{k}-\mathbf{q}}\right]\right\rangle \tag{19}$$

3. LONG-WAVELENGTH LIMIT

In the long-wavelength $(k \to 0)$ limit, the elements of the frequency moments are as follows:

$$\hat{\Omega}_2^{11}(\mathbf{k}) = \hat{\Omega}_2^{22}(\mathbf{k}) = \hat{\Omega}_2^{33}(\mathbf{k}) = \omega_p^2\left\langle\gamma^{-1}\left(1 - \frac{v^2}{3c^2}\right)\right\rangle$$

$$\hat{\Omega}_3^{12}(\mathbf{k}) = -\hat{\Omega}_3^{21}(\mathbf{k}) = \mathrm{i}\omega_p^2\Omega\left\langle\gamma^{-2}\left(1 - \frac{2}{3}\frac{V^2}{c^2}\right)\right\rangle\cos\theta$$

$$\hat{\Omega}_4^{11}(\mathbf{k}) = \omega_p^4\left\{1 + \frac{k^2}{\kappa^2}\left(1 - \frac{2}{15}\beta_p E_{\mathrm{corr}}\right) + \frac{\beta_p}{\beta_r^4\eta\hbar^3c^3}\right.$$
$$\left.\times\left[\frac{\pi^2}{45} + \frac{1}{3\pi^2}G_0 + \frac{1}{30\pi^2}(5G_1 + 2G_2)k^2\right]\right\}\left\langle\gamma^{-2}\left(1 - \frac{2v^2}{3c^2}\right)\right\rangle$$

$$\hat{\Omega}_4^{13}(\mathbf{k}) = \hat{\Omega}_4^{31}(\mathbf{k}) = 0$$

$$\hat{\Omega}_4^{22}(\mathbf{k}) = \omega_p^4\left\{1 + \frac{k^2}{\kappa^2}\left(1 - \frac{2}{15}\beta_p E_{\mathrm{corr}}\right) + \frac{\beta_p}{\beta_r^4\eta\hbar^3c^3}\right.$$
$$\left.\times\left[\frac{\pi^2}{45} + \frac{1}{3\pi^2}G_0 + \frac{1}{30\pi^2}(5G_1 + 2G_2)k^2\right]\right\}\left\langle\gamma^{-2}\left(1 - \frac{2v^2}{3c^2}\right)\right\rangle \tag{20}$$

$$\hat{\Omega}_4^{33}(\mathbf{k}) = \omega_p^4\left\{1 + \frac{k^2}{\kappa^2}\left(3 + \frac{4}{15}\beta_p E_{\mathrm{corr}}\right) + \frac{\beta_p}{\beta_r^4\eta\hbar^3c^3}\right.$$
$$\left.\times\left[\frac{\pi^2}{45} + \frac{1}{3\pi^2}G_0 + \frac{1}{30\pi^2}(5G_1 + G_2)k^2\right]\right\}\left\langle\gamma^{-2}\left(1 - \frac{2v^2}{3c^2}\right)\right\rangle$$

$$\hat{\Omega}_5^{12}(\mathbf{k}) = -\hat{\Omega}_5^{21}(\mathbf{k}) = i2\omega_p^4\Omega\left\{1 + \frac{1}{2}\left(3 - \frac{2}{15}\beta_p E_{\mathrm{corr}}\right)\frac{k^2}{\kappa^2} + \frac{\beta_p}{2\beta_r^4\eta\hbar^3c^3}\right.$$
$$\left.\times\left[\frac{\pi^2}{9} + \frac{5}{3\pi^2}G_0 + \frac{1}{30\pi^2}(25G_1 + 9G_2)k^2\right]\right\}\langle\gamma^{-5}\rangle\cos\theta$$

$$\hat{\Omega}_5^{23}(\mathbf{k}) = -\Omega_5^{32}(\mathbf{k}) = i2\omega_p^4\Omega\left\{1+\frac{1}{2}\left(6+\frac{3}{15}\beta_p E_{\text{corr}}\right)\frac{k^2}{\kappa^2}+\frac{\beta_p}{2\beta_r^4\eta\hbar^3c^3}\right.$$
$$\left.\times\left[\frac{\pi^2}{9}+\frac{5}{3\pi^2}G_0+\frac{1}{30\pi^2}(25G_1+8G_2)k^2\right]\right\}\langle\gamma^{-5}\rangle\sin\theta$$

where E_{corr}, G_0, G_1, G_2, κ^2, and Ω are defined as follows:

$$E_{\text{corr}} = \frac{\eta}{2V}\sum\frac{4\pi e^2}{q^2}g_{\mathbf{q}} \qquad (V \text{ is the volume of the system})$$

$$G_0 = \int dx\, x^2 f(x)\eta g_{\mathbf{q}}$$

$$G_1 = \int dx\, x^2 f(x)\frac{1}{q}\frac{\partial}{\partial q}\eta g_{\mathbf{q}}$$

$$G_2 = \int dx\, x^2 f(x)\frac{\partial^2}{\partial q^2}\eta g_{\mathbf{q}} \tag{21}$$

$$\kappa^2 = 4\pi e^2\eta\beta_p \qquad (\eta \text{ is the particle density})$$

$$\Omega = \frac{eB^0}{mc} \qquad (\text{the electron cyclotron frequency})$$

4. RELATIVISTIC EFFECTS ON PLASMA DISPERSION

As mentioned in the introduction, the high-frequency modes of interest are the "ordinary" and the "extraordinary" modes. The extraordinary mode of interest is the one with cutoff frequency

$$\omega_2 = \frac{\Omega}{2}\left[1+\left(1+\frac{4\omega_p^2}{\Omega^2}\right)^{1/2}\right]$$

these modes are considered when the direction of propagation is along and perpendicular to the magnetic field. We use a coordinate system where $\mathbf{k} = (0, 0, k)$ and $\mathbf{B}^0 = (B_x^0, 0, B_z^0)$, i.e., the k system.

4.1. Ordinary Mode

For parallel propagation the ordinary mode does not exist; instead, we have a longitudinal mode which oscillates at the plasma frequency. The dispersion relation which determines the behavior of longitudinal plasmons is given by

$$\varepsilon_{33}(\mathbf{k}) \equiv 1+\alpha_{33}(\mathbf{k}\omega) = 0 \tag{22}$$

After applying a small perturbation (Kalman and Genga, 1986; Genga, 1988*a*,*b*) to the dispersion relation, we find that the ensuing plasmon frequency is given by

$$\omega^2(\mathbf{k}) = \omega_p^2\left(1 + C_L + A_L\frac{k^2}{\kappa^2}\right)\left\langle \gamma^{-2}\left(1 - \frac{2v^2}{3c^2}\right)\right\rangle \tag{23}$$

where

$$A_L(\gamma, \beta_r) = 3 + \frac{4}{15}\beta_p E_{\text{corr}} + \frac{1}{30\pi^2}\frac{\beta_p\kappa^2}{\beta_r^4\eta\hbar^3c^3}(5G_1 + G_2) \tag{24}$$

$$C_L(\gamma, \beta_r) = \frac{\beta_p}{\beta_r^4\eta\hbar^3c^3}\left(\frac{\pi^2}{45} + \frac{1}{3\pi^2}G_0\right) \tag{25}$$

For propagation perpendicular to the magnetic field, the ordinary mode exists and the dispersion relation which determines its behavior is given by

$$(\varepsilon_{11} - n^2) = 0 \tag{26}$$

The ensuing frequency of the ordinary mode in this case can be written as

$$\omega^2 = \omega_p^2\left[1 + C_L + \left(\frac{c^2}{\omega_p^2} + A_T\right)k^2\right]\left\langle \gamma^{-2}\left(1 - \frac{2v^2}{3c^2}\right)\right\rangle \tag{27}$$

where

$$A_T(\gamma, \beta_r) = 1 - \frac{2}{15}\beta_p E_{\text{corr}} + \frac{\beta_p\kappa^2}{30\pi^2\beta_r^4\eta\hbar^3c^3}(5G_1 + 2G_2) \tag{28}$$

4.2. Extraordinary Mode

For propagation along the magnetic field the dispersion relation which determines the behavior of extraordinary mode is

$$(\varepsilon_{11} - n^2)^2 - \varepsilon_{12}^2 = 0 \tag{29}$$

This leads to the ensuing expression for the frequency of the extraordinary mode:

$$\omega^2(\mathbf{k}) = \omega_2^2\left[1 + \frac{\omega_p^2}{\omega_2^2}C_L + \frac{1}{\omega_p^2}\left(c^2 + \frac{\omega_p^4}{\omega_2^2}A_T\right)k^2\right]\left\langle \gamma^{-2}\left(1 - \frac{2v^2}{3c^2}\right)\right\rangle \tag{30}$$

In the case of propagation perpendicular to the magnetic field we find that the dispersion is given by

$$(\varepsilon_{22} - r^2)\varepsilon_{33} - \varepsilon_{33}^2 = 0 \tag{31}$$

which leads to the frequency

$$\omega^2(\mathbf{k}) = \omega_2^2\left[1 + \frac{\omega_p^2}{\omega_2^2}C_L + \frac{1}{2\omega_p^2}\left(c^2 + \frac{2\omega_p^4}{\omega_2^2}\frac{A_x}{\kappa^2}\right)k^2\right]\left\langle \gamma^{-2}\left(1 - \frac{2v^2}{3c^2}\right)\right\rangle \tag{32}$$

where

$$A_x = 2 + \frac{1}{15}\beta_p E_{\rm corr} + \frac{\beta_p \kappa^2}{60\pi^2 \beta_r^4 \eta \hbar^3 c^3}(10G_1 + 3G_2) \tag{33}$$

5. CONCLUSION

The frequency of the relativistic modes in consideration are reduced by a factor $\langle \gamma^{-1}(1 - v^2/3c^2)\rangle$ as in the magnetic field free case (Genga, 1988b). The effects of finite radiation temperature on correlations remain the same as in the results based on the nonrelativistic approach (Genga, 1988a).

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